Review of Biomedical Instrumentation

COLUMBIA | ENGINEERING
The Fu Foundation School of Engineering and Applied Science

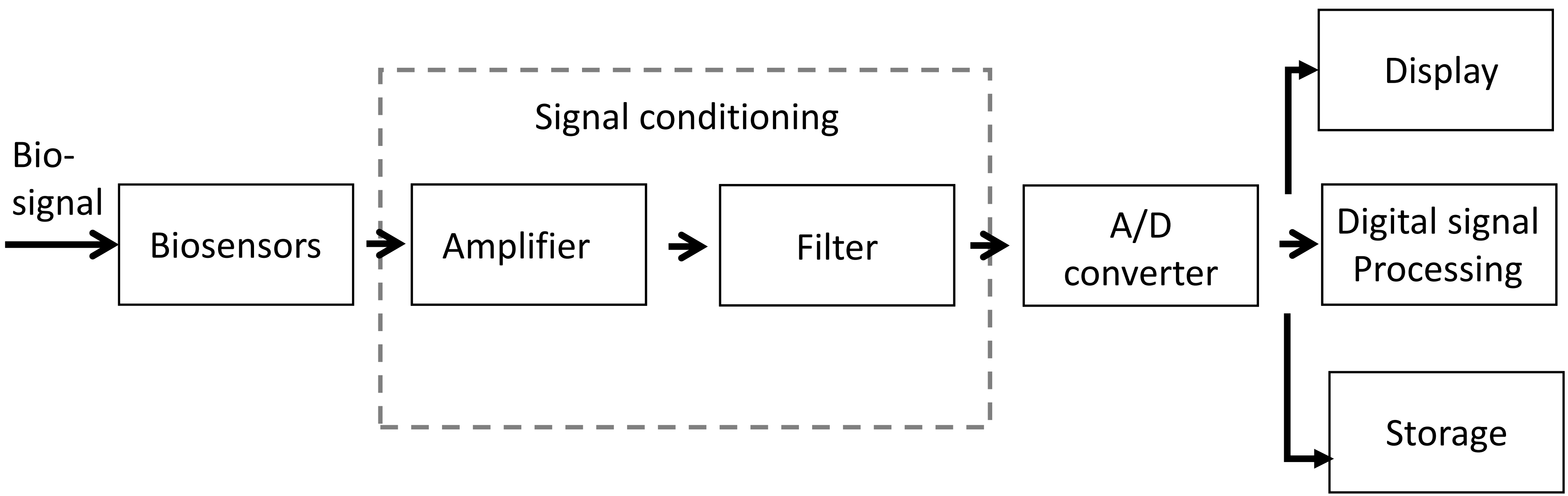
Module final: Nov. 5, 2025

- You can bring:
  - The Laplace transform table (with no notes)
  - A simple calculator with trigonometric functions
  - Smart phone **NOT** allowed
  - Cheat-sheet **NOT** allowed

- Office hours
  - Oct. 30 (Thursday): 11-1pm, BME multi-purpose room Mudd 343
  - Nov. 4 (Tuesday): 10-12pm, BME conference room ET353

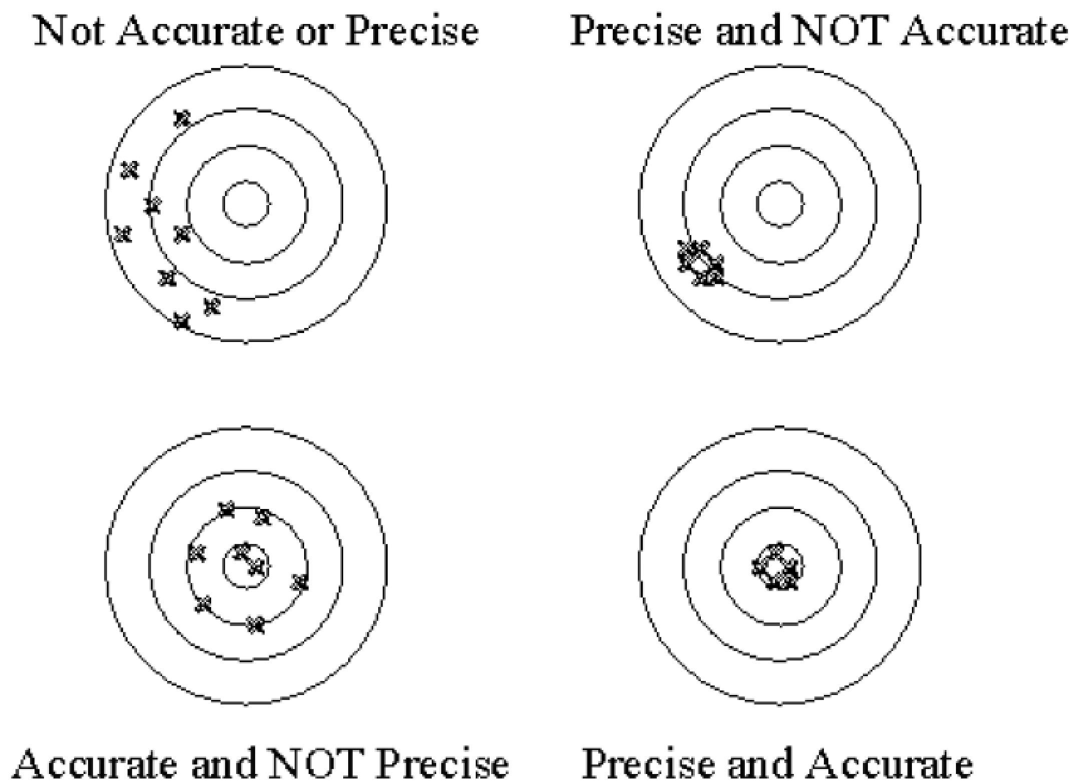
Biomedical instrumentation

- Multiple stages



Accuracy & precision

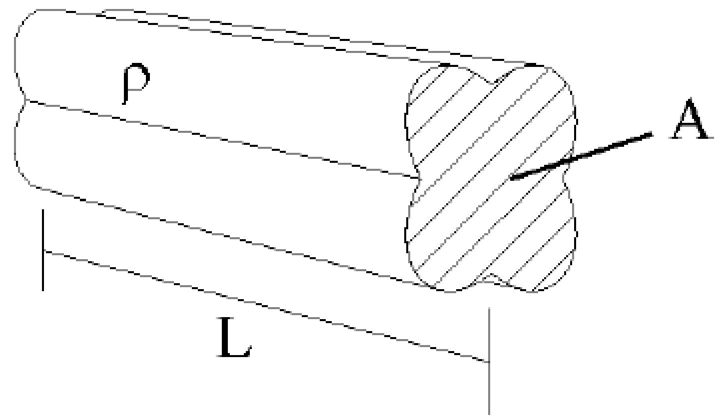
- Accuracy refers to the difference between the true value and the actual value measured by the sensor.
- Precision refers to the degree of measurement reproducibility



Variable-resistive sensors

- The resistance of an electrical conductor is

$$R = \frac{\rho L}{A}$$



Where:

R is resistance
L is the length of the conductor
A is the cross-sectional area of the conductor
P is the resistivity of the material

Strain gages

- One of the most common displacement transducer (strain -> change in resistance)
- Strain, or axial strain, is defined as the fractional change in length in response to applied force:

$$\varepsilon = \frac{\Delta L}{L}$$

Strain gages

- What the relationship between ΔR and ε?

$$dR = d(\frac{\rho L}{A}) = \frac{\rho}{A}dL + \frac{L}{A}d\rho - \frac{\rho L}{A^2}dA$$

$$\Downarrow$$

$$\frac{dR}{R} = \frac{dL}{L} + \frac{d\rho}{\rho} - \frac{dA}{A}$$

Strain gages

- Poisson's ratio defines the relationship between the change in diameter of a body experiencing uniaxial stress and change in length

$$\nu = -\frac{\Delta D}{D} / \frac{\Delta L}{L}$$

Minus sign means increase in diameter resulting in decrease in length

Strain gages

- For a circular wire, the cross-sectional area

$$A = \pi(\frac{D}{2})^2$$

$$\Delta A = d(\pi(\frac{D}{2})^2) = \pi(\frac{D}{2})dD$$

$$\frac{\Delta A}{A} = -2\nu\frac{\Delta L}{L}$$

Strain gages

$$\frac{\Delta R}{R} = (1 + 2\nu)\frac{\Delta L}{L} + \frac{\Delta \rho}{\rho}$$

Dimensional effect

Piezoresistive effect: changes in the lattice structure of the material induced by strain

Strain gages

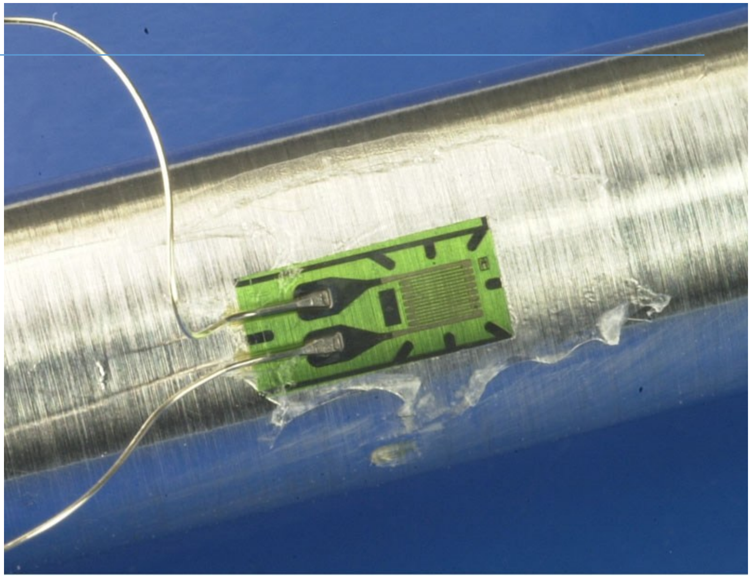
- The gage factor G

$$G = \frac{\Delta R / R}{\Delta L / L} = (1 + 2\nu) + \frac{\Delta\rho / \rho}{\Delta L / L}$$

- The gage factor for most metals is primarily a function of dimensional effects, i.e. $G = (1 + 2\nu)$

$$\varepsilon = \frac{1}{G}\frac{\Delta R}{R}$$

Strain gages



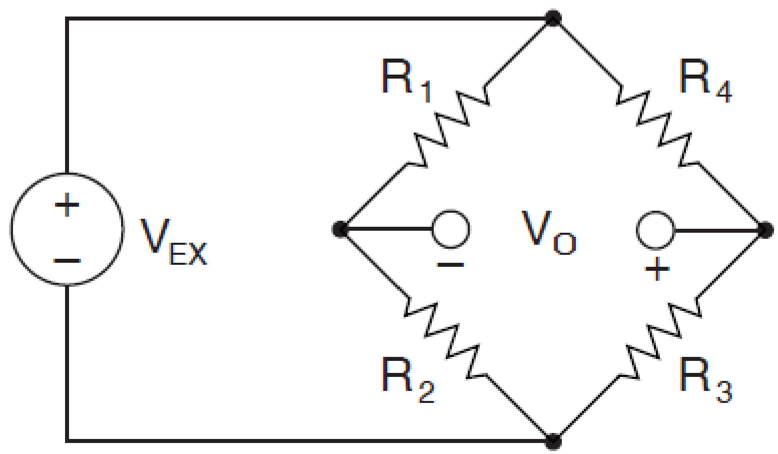
- The strain and change in resistance

$$\varepsilon = \frac{\Delta L}{L} = \frac{1}{G}\frac{\Delta R}{R}$$

- Strain of material is typically very small
- A typical strain gage has G = 2, R=300Ohm, given a strain of 0.000001, the change in resistance is

$$\Delta R = \varepsilon G R = 600\,\mu\Omega$$ Too small to measure with a multimeter!

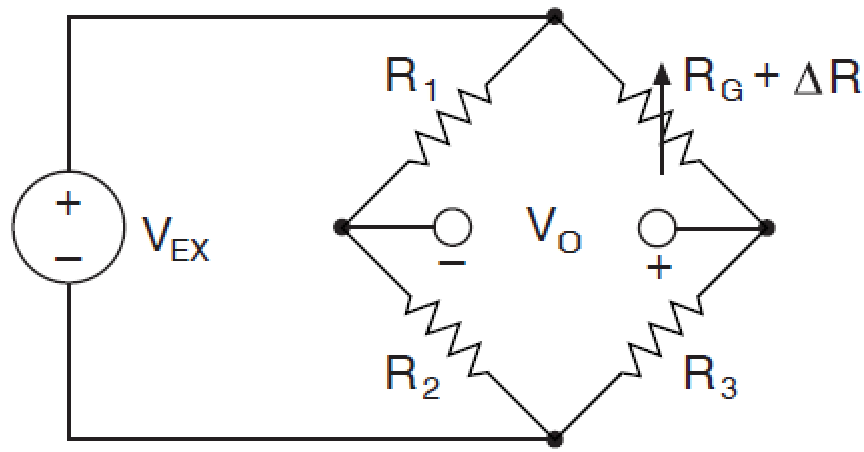
Wheatstone Bridge



$$V_O = \left(\frac{R_3}{R_3 + R_4} - \frac{R_2}{R_1 + R_2}\right)V_{EX}$$

- If $R_4/R_3 = R_1/R_2$, the Vo =0, which means the bridge is balanced.

Quarter Wheatstone Bridge

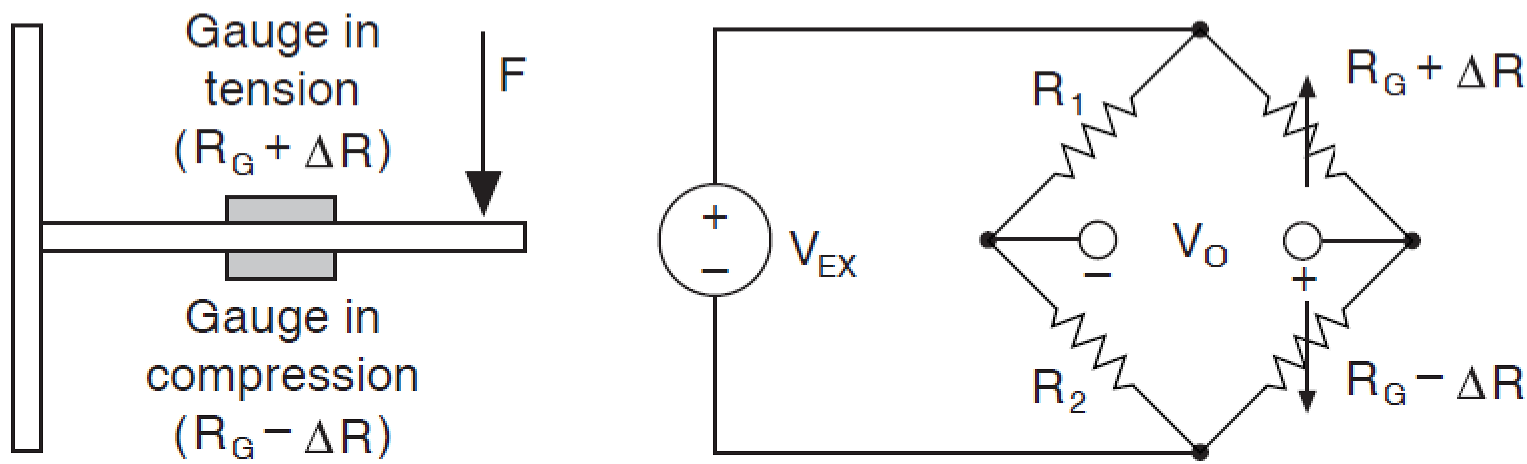


- If $R_1 = R_2$, and $R_3 = R_{G,}$

$$V_0 = -\frac{G \cdot \varepsilon}{4} \frac{1}{1 + G \cdot \varepsilon/2} V_{EX}$$

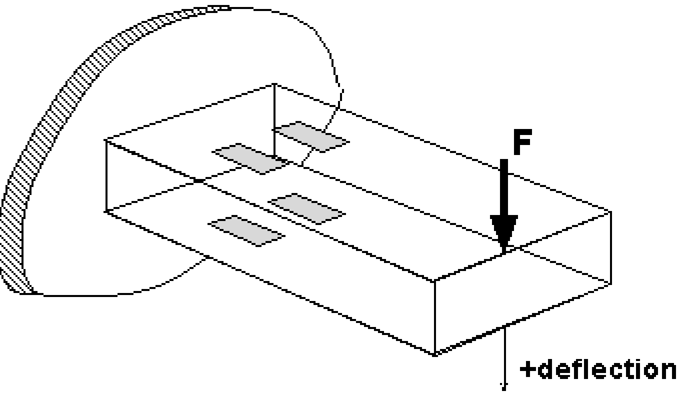
Where G is gage factor

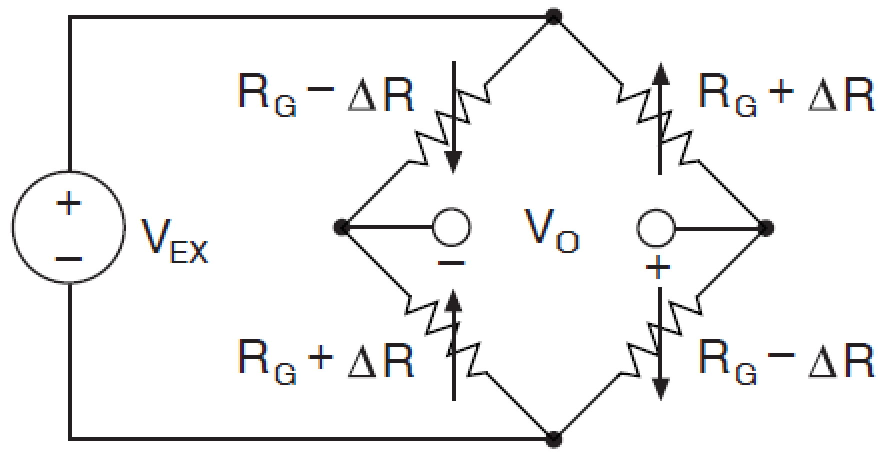
Half Wheatstone Bridge



$$V_0 = -\frac{G \cdot \varepsilon}{2} V_{EX}$$

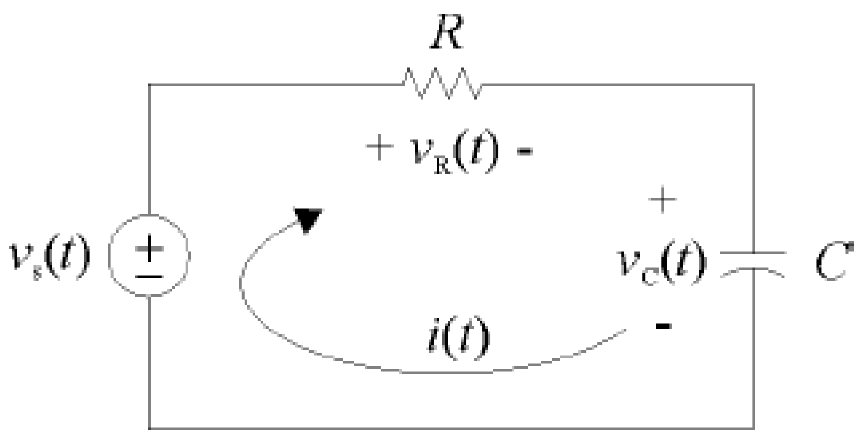
Full Wheatstone Bridge





$$V_0 = -G \cdot \varepsilon \cdot V_{EX}$$

Circuit analysis in Laplace domain



$$V_s(t) = RC\,\frac{dV_c(t)}{dt} + V_c(t)$$

$$V_s(t) = u(t)$$

Laplace transform

$$V_c(s) = \frac{1}{1+RCs}\cdot\frac{1}{s}$$

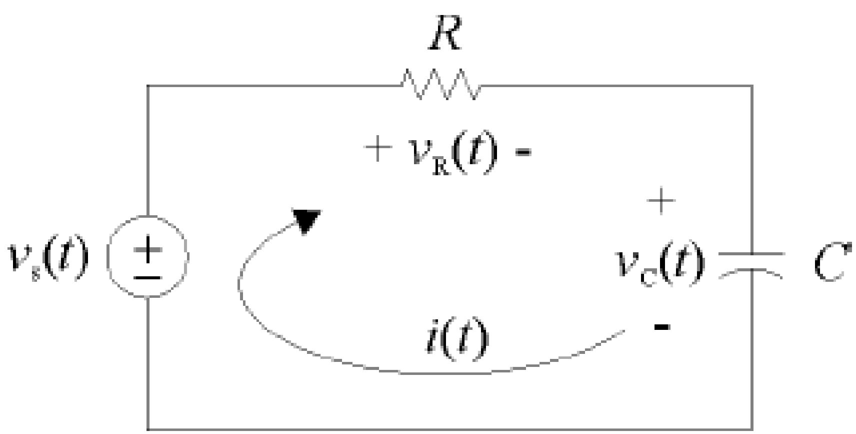
Inverse Laplace transform

$$V_c(t) = 1 - e^{-\frac{t}{RC}}$$

Noise

- Any unwanted signal that corrupts the signal of interest.
- Common types of noise are:
  - Extraneous noise
  - Thermal noise (white noise)
  - power noise
  - 1/f noise (pink noise)

Low pass filter



- Input signal:

$$v_s(t) = \sin(t) + \sin(100t)$$

- What is the voltage across the capacitor?

$$V_c(s) = \frac{1}{1+RCs} V_s(s)$$

$$V_s(s) = \frac{1}{s^2+1} + \frac{100}{s^2+100^2}$$

Low-pass filter

$$V_c(s) = \frac{1}{1+RCs}\cdot\frac{1}{s^2+1} + \frac{1}{1+RCs}\cdot\frac{100}{s^2+100^2}$$

Response to sinusoidal input with a freq. of 1 rad/s

Response to sinusoidal input with a freq. of 100 rad/s

Low-pass filter

- Response to the 1 rad/s sinusoidal signal

$$V_{c1}(s) = \frac{1/RC}{s + 1/RC} \cdot \frac{1}{s^2+1}$$

$$L^{-1}[V_{c1}(s)] = \frac{1}{RC}\left(\frac{1}{(\frac{1}{RC})^2+1} e^{-\frac{t}{RC}} + \frac{1}{\sqrt{(\frac{1}{RC})^2+1}} \sin(t-\theta)\right)$$

$$= \frac{RC}{(RC)^2+1} e^{-\frac{t}{RC}} + \frac{1}{\sqrt{(RC)^2+1}} \sin(t-\theta)$$

Where: $\theta = \tan^{-1}(RC)$

Transient response

Steady-state response

Low-pass filter

- Response to the 100 rad/s sinusoidal signal

$$V_{c2}(s) = \frac{1/RC}{s + 1/RC} \cdot \frac{100}{s^2 + 100^2}$$

$$L^{-1}[V_{c2}(s)] = \frac{1}{RC}\left(\frac{100}{(\frac{1}{RC})^2 + 100^2} e^{-\frac{t}{RC}} + \frac{1}{\sqrt{(\frac{1}{RC})^2 + 100^2}} \sin(100t - \theta)\right)$$

$$= \frac{100RC}{(100RC)^2 + 1} e^{-\frac{t}{RC}} + \frac{1}{\sqrt{(100RC)^2 + 1}} \sin(100t - \theta)$$

Where: $\theta = \tan^{-1}(100RC)$

Low-pass filter

$$v_s(t) = \sin(t) + \sin(100t)$$

- If RC = 1

$$V_{c1}(t) = \frac{1}{\sqrt{(RC)^2 + 1}} \sin(t - \theta) = \frac{1}{\sqrt{2}} \sin(t - \theta)$$

$$V_{c2}(t) = \frac{1}{\sqrt{(100RC)^2 + 1}} \sin(100t - \theta)$$

$$= \frac{1}{\sqrt{10001}} \sin(100t - \theta)$$

High frequency signal was attenuated by 99% while the low frequency signal was attenuated by 30%

Discrete Fourier Transform

- DFT is necessary to analyze and represent discrete signals in the frequency domain.

$$X(m) = \sum_{k=0}^{N-1} x(k) e^{-j\frac{2\pi mk}{N}}; m = 0, 1, \ldots, N-1$$

Where the index m represents the digital frequency index, x(k) is the sampled data of x(t), and N is an even number that represents the number of samples for x(k)

Fast Fourier Transform (FFT)

- DFT needs $O(N^2)$ operations because it has N/2 outputs X_k, and each output requires a sum of N terms
- If N is a power of 2, some tricks can be used compute the same result with only $(N/2)\log_2(N)$ operations. This is called Fast Fourier Transform, which is significantly faster.

Estimation issues

- The highest frequency that can be estimated is determined by the rate at which the signal was sampled and is given by

$$f_{\max} = \frac{1}{2T_s}$$

Where T_s is sampling interval

Estimation issues

- The frequency resolution of spectral estimates is also determined by the segment length and the sampling frequency

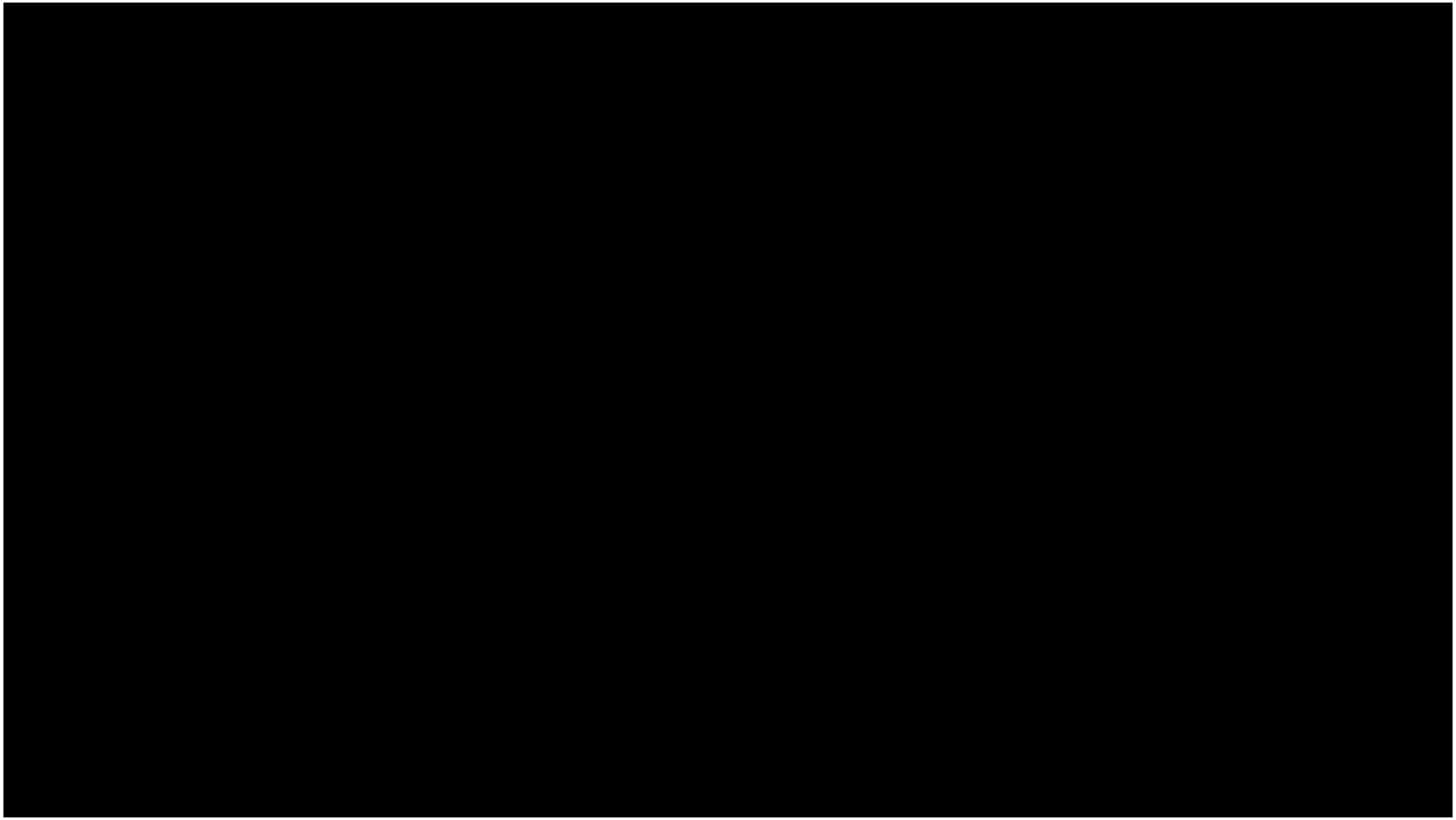
$$\Delta f = \frac{f_s}{N}$$

Where f_s is sampling frequency, and N is the length of data

Estimation issues

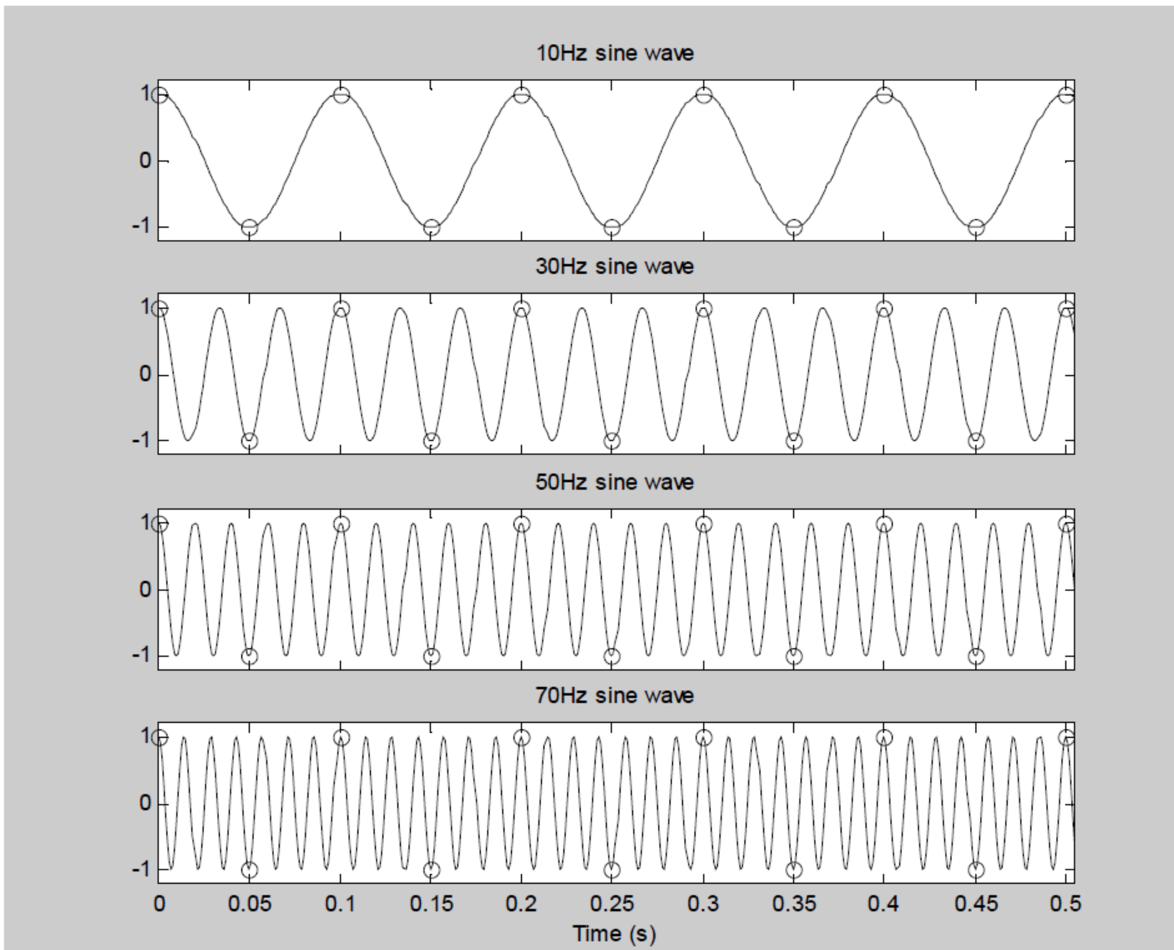
- As with the Fourier transform, record length selection is particularly important when computing the spectra of periodic signals; it is best to choose a record length that yields a frequency increment that divides evenly into the signal's fundamental frequency.
- For FFT, since the number of samples must be a power of 2, it sometimes may lead to estimates at strange frequencies.

Wagon-wheel effect



Aliasing

- Aliasing is the confusion of high- and low-frequency components in the original (analog) signal



Aliasing

- For a signal sampled at a rate of *fs* samples/second, the frequencies given by

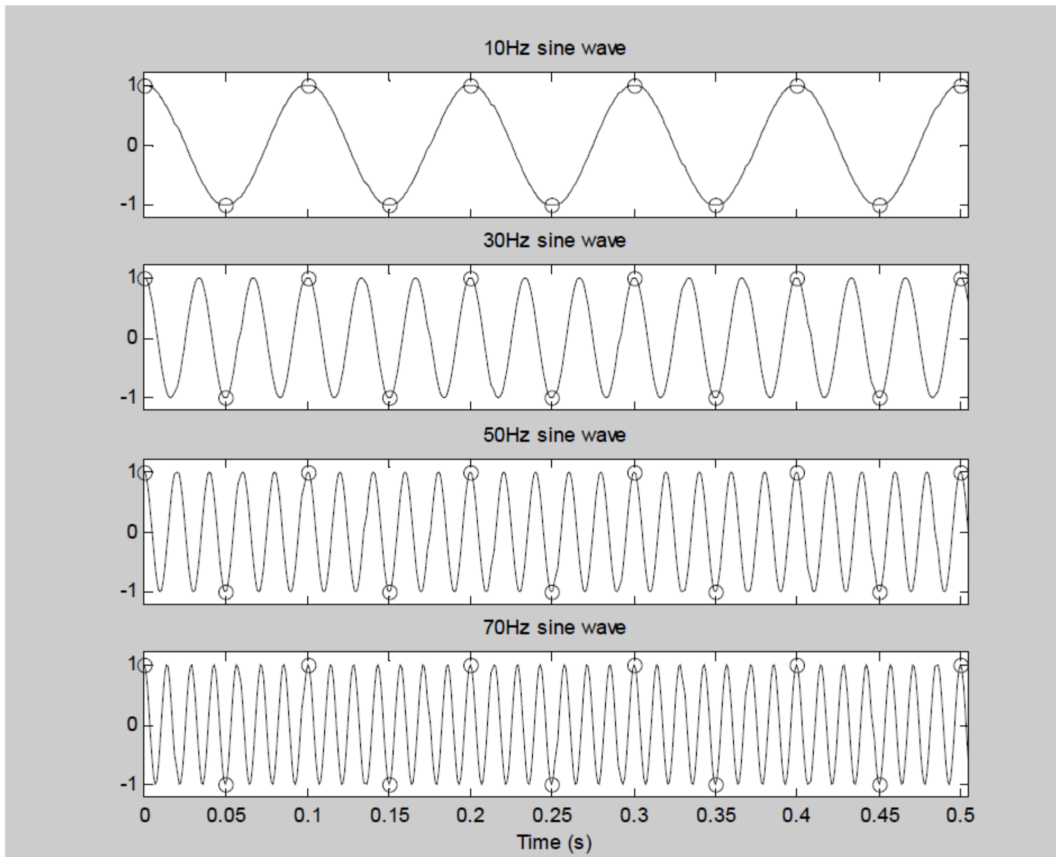
$$f_i = nf_s \pm f \quad \text{for } n = 1, 2, 3, \ldots$$

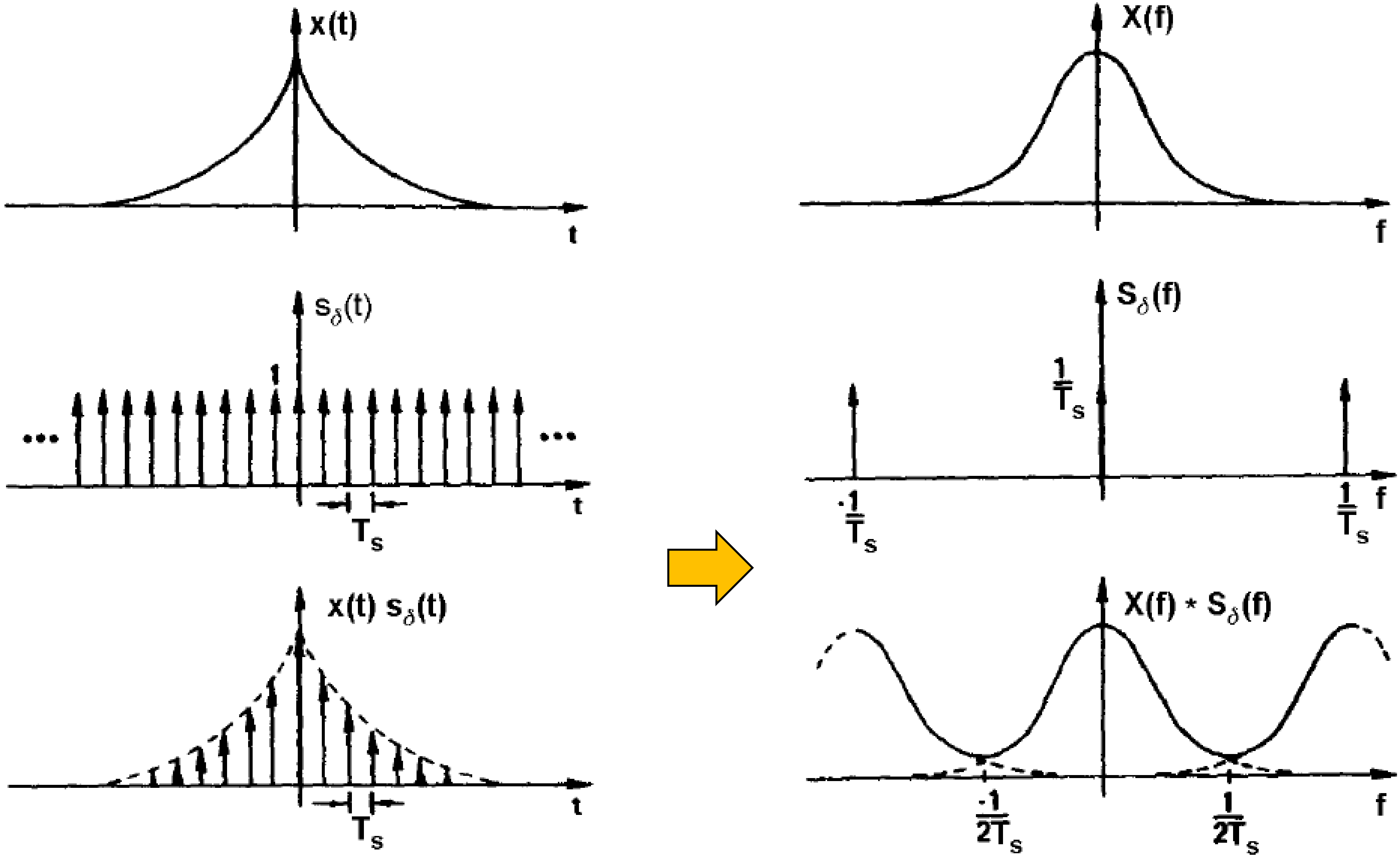
$$\text{where } 0 \le f \le \frac{f_s}{2}$$

cannot be distinguished from each other.

Aliasing

- With a sampling rate of 20 Hz, components with frequencies of 10, 30, 50, 70, etc. will all yield exactly the same sample values, since they are aliased to 10 Hz.





x(t)
t
s_δ(t)
1
...
T_s
x(t) s_δ(t)
X(f)
f
S_δ(f)
1/T_s
-1/T_s
X(f) * S_δ(f)
-1/2T_s
1/2T_s